Traffic safety at intersections: a priority based approach for cooperative collision avoidance

Gabriel R. Campos[1,2], Paolo Falcone[2], Jonas Sjöberg[2]

[1] *DEIB, Politecnico di Milano, Italy*
Email: `gabriel.rodriguesdecampos@polimi.it`
[2] *Department of Signals and Systems, Chalmers University of Technology, Sweden.*
Email: `paolo.falcone,jonas.sjoberg@chalmers.se`

Abstract: In this paper, we consider the coordination problem of multiple autonomous vehicles at traffic intersections. In particular, we exploit a cooperative, sequential conflict resolution approach based on a pre-defined decision order. Using an optimal control formulation, we show how coordination can be ensured by solving two local problems where collision avoidance is enforced as time-dependent state constraints. We will analyse the feasibility of a given sequence with respect to different decision criteria and present simulation results supporting our results.

Keywords: Safety systems, Conflict resolution, Cooperative control, Autonomous driving

1. INTRODUCTION

The development of new Intelligent Transportation Systems (ITS) has enabled safer, smarter, and greener transport systems [Behere et al. (2013)]. Recent research has been focusing, among others, in prevention and mitigation of accidents, reduction of greenhouse gas emissions and efficiency in terms of energy and infrastructure utilization. Such efforts are fuelled by alarming statistics on road accident fatalities and the rapidly growing number of vehicles on the road. According to the World Health Organization, 1.24 million people died in traffic accidents during 2013 and as many as 50 million people suffered non fatal injuries, and this number can increase up to 1.9 million by 2020 if no action is taken [World Health Organisation (2013)]. Even if fatalities and injuries numbers render safety more compelling than efficiency, the effects of inefficient road transportation (e.g., traffic congestions, pollutants, greenhouse gasses and fuel consumption) on the environment, health and finance are also significant. For instance, road transportation is currently responsible for 16.5 percent of the anthropogenic greenhouse gas emissions [International Energy Association (2013)], and congestion locks down most major cities during rush hours. According to estimates by the U.S. Treasury, 7 billion liters of gas is wasted through congestion in the U.S. alone, which together with wasted time and productivity incurs a cost of over 100 billion dollars annually [U.S. Department of Treasury (2012)].

A particular interesting problem, both from a safety and efficiency point of view, is collision avoidance at traffic intersections [Hafner et al. (2013); Doerzaph et al. (2008); Alexander et al. (2011)]. In Europe, intersections-related accidents are responsible for 21% of traffic related deaths and 43% of the non-fatal injuries [Simon et al. (2009)]. Similar numbers have been reported from the U.S. [National Traffic Highway Safety Association (2010)]. Due to the high risk of accidents, these traffic scenarios are among the most regulated ones, with vehicles guided simultaneously by traffic lights, signs, road-markings and right-of-way rules. As a consequence, they often form bottlenecks in the traffic system and even when not causing congestion, existing coordination rules are inherently inefficient, enforcing unnecessary decelerations and stops and thereby wasting both fuel and time.

Cooperative ITS have the potential to improve traffic flow and safety near intersections, without relying on inefficient traffic lights or error-prone human control. Instead, vehicles equipped with communication devices, have to coordinate and agree on how to cross the intersection without collisions. Informally, the *coordination problem* amounts to deciding the control functions for the individual vehicles that allow them to safely reach their destination. It consequently entails avoiding both collisions and traffic deadlocks, and doing so in a manner compatible with the physical capabilities and constraints of the vehicles. Note that the coordination problem has, in general, infinitely many solutions corresponding to different control functions and different crossing orders, i.e., temporal orders under which the vehicles occupies the critical regions. In today's traffic system, the strategy resulting from the interplay between human drivers, signal infrastructure and traffic rules gives *one* of the many solutions. However, as noted in the introduction, this particular strategy suffers from inefficient performance, and can, due to the presence of human drivers, unintentionally lead to constraint violations (collisions). Given a performance metric, we can also

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set an *optimal coordination problem*, where the best of all feasible solutions to the coordination problem is sought.

Cooperative conflict resolution problems for autonomous vehicles at road intersections is a subject that has attracted a lot of research efforts recently. For instance, several works focused on the coordination problem based on the multi-agent systems paradigm and a rule-based approach [Dresner and Stone (2004, 2005, 2008); Kowshik et al. (2011)]. Other works, instead, used Model Predictive Control (MPC) coordination strategies [Kim and Kumar (2014); Hult et al. (2015); Campos et al. (2013, 2014)]. For instance, [Hult et al. (2015)] exploits the structure of the centralized, finite time optimal control problem, in order to propose an approximate solution, while [Campos et al. (2013)] considered a fully decentralized solution to the intersection conflict resolution problem, based on sub-optimal decision-making heuristics, using the concept of *decision sequence*. This results were later extended in [Campos et al. (2014)], where authors proposed a low complexity receding horizon control framework. It is worth mentioning that collision avoidance has also been approached from an active safety point of view, where the driver is overridden in case safety is compromised. Among others, [Hafner et al. (2011, 2013)] exploited hybrid systems theory and [Ahn et al. (2014); Bruni et al. (2013); Colombo and Del Vecchio (2015); Colombo and Del Vecchio (2012)] a scheduling-based approach, where the equivalence between the verification problem and the feasibility of a scheduling problem is presented.

In this paper, we consider a fully decentralized solution to the intersection conflict resolution problem, suitable for fully autonomous vehicles (contrary to [Hafner et al. (2011, 2013)], which focus on intervention). We abstract from the (many) implementation issues and focus on the fundamental aspects of the underlying decision making problems. Our solution relies on a cooperatively pre-determined decision order (enabling sequential decision making)for the computation of optimal collision-free trajectories (contrary to [Colombo and Del Vecchio (2012)], which focuses on feasible crossings sequences and not optimality). This work complements our previous results [Campos et al. (2013, 2014)], by considering general decision orders and providing a comparison in terms of feasibility for different decision criteria.

The rest of this paper is organized as follows. Section 2 presents the problem formulation while Section 3 addresses the centralized control formulation. In Section 4, a sequential and decentralized approach is presented, where formal feasibility conditions are derived for a given decision order. Finally, Section 5 presents some simulations results supporting the theoretical contributions of this paper and Section 6 some final conclusions.

2. SYSTEM DESCRIPTION

Consider $N > 1$ autonomous vehicles/agents approaching a traffic intersection as shown in Figure 1. For each agent i, we assume that:

- a path is given and is known;
- the assigned path is perfectly followed;
- the acceleration along the path can be varied;

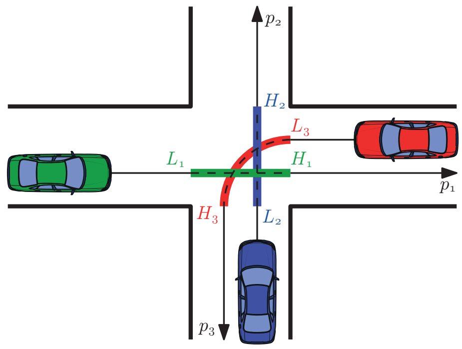


Fig. 1. Illustration of the considered scenario. Several autonomous vehicles approach an intersection defined by a range of positions over pre-defined paths. Vehicles are supposed to approach the intersection with a desired speed, where the variable of control is the longitudinal acceleration.

- all vehicles have synchronized clocks and are located before the intersection at the initial time instant.

Let $x_i = [p_i \ v_i]^T \in X_i = P_i \times V_i$ denote the state of each vehicle $i \in \mathcal{N} = \{1, \ldots, N\}$, where $p_i \in P_i$, $v_i \in V_i$ and P_i and V_i represent the sets of all admissible (scalar) longitudinal positions and velocities along the path, respectively. Each agent is modeled as a discrete time double integrator

$$x_i(t+1) = A\ x_i(t) + B\ u_i(t), \tag{1}$$

where $A = [1\ 1; 0\ 1]$ and $B = [0\ 1]^T$. Furthermore, we assume that a full measurement of the state $x_i(t)$ is available at all times. Throughout the rest of the article, t is considered to be the current time such that $(\cdot)(t+k)$ denotes the predicted value of variable $(\cdot)$ at time $t+k$, computed at time t.

As a part of the assigned driving task, each agent i has a given reference (i.e., desired) velocity denoted by $v_{di} \in V_i$. Furthermore, let $x = [x_i^T, \ldots, x_N^T]^T, y = [y_i^T, \ldots, y_N^T]^T, u = [u_i^T, \ldots, u_N^T]^T, v_d = [v_{di}, \ldots, v_{dN}]^T$ denote the state, the input, the output, and the desired velocity vector, respectively, for the entire system composed of N vehicles. The following constraints are also taken into account:

(1) ***Actuator limitations***

To ensure that the control input u_i (longitudinal acceleration) is within the admissible actuator range, each vehicle is assumed to be subject to:

$$u_i^{\min} \ \leq \ u_i(t) \ \leq \ u_i^{\max}, \qquad \forall t \ \geq 0, \tag{2}$$

which yields $U_i = \{\ u_i |\ u_i \in [u_i^{\min}, u_i^{\max}]\ \}$.

(2) ***State constraints***

Vehicles' velocities are constrained such that

$$0 < \ v_i^{\min} \leq \ v_i(t) \ \leq v_i^{\max}, \qquad \forall t \ \geq 0, \tag{3}$$

which yields $V_i = \{\ v_i |\ v_i \in [v_i^{\min}, v_i^{\max}]\ \}$.

(3) ***Safety constraints***

The proposed collision avoidance solution relies on the design local controllers preventing a given vehicle of accessing the intersection if it is already occupied by any other vehicle. We introduce the following definitions.

Definition 1. (**Critical set**). For each agent $i \in \mathcal{N}$, let Cr_i denote the *critical set*, i.e., the set of all positions along the path where a collision is possible and defined as

$$\mathrm{Cr}_i \triangleq \{\ x_i \in X_i |\ p_i \in [L_i, H_i]\ \}, \tag{4}$$

where $L_i < H_i$ are bounds on the position along the path of vehicle i defining the intersection. Note that these parameters are dependent on the geometry of the workspace and are time-invariant.

Definition 2. (**Occupancy interval**). For each agent $i \in \mathcal{N}$, the *occupancy interval* of the intersection for a given predicted control sequence can be expressed as

$$\Gamma_{i,t}\left(x_i(t), u_i(t), u_i(t+1), \ldots\right) = \{k |\ x_i(k) \in \mathrm{Cr}_i\}, \tag{5}$$

where $x_i(t+1)$ is given by (1) and $\{u_i(t), u_i(t+1), \ldots\}$ denotes a control sequence. In order to simplify the notation, we will consider throughout the rest of the paper $\Gamma_{i,t}$ as the shorthand form of $\Gamma_{i,t}\left(x_i(t), u_i(t), u_i(t+1), \ldots\right)$.

From Def. 2 the following collision avoidance constraint follows

$$\Gamma_{i,t} \cap \Gamma_{j,t} = \emptyset, \forall\ i, j \in \mathcal{N}, j \neq i. \tag{6}$$

For the sake of clarity, if at time t the condition $p_i(t) < L_i$ holds, we will state that agent i is "before" the critical set, while if $p_i(t) > H_i$ holds we will say that the agent is "after" the critical set. Finally, we introduce the polytopes Ω_i and Υ_i as the set of states corresponding to the vehicle being before and after the intersection, respectively, and are defined as

$$\Omega_i = \{x_i |\ v_i^{\min} \leq v_i \leq v_i^{\max},\ 0 \leq p_i \leq L_i\},$$

and $\Upsilon_i = X / \{\mathrm{Cr}_i \cup \Omega_i\}$, see Figure 3.

3. CENTRALIZED PROBLEM FORMULATION

Consider the following global cost function

$$J_{\mathrm{centr},t} = \sum_{k=0}^{\infty} \|v(t+k) - v_d\|_Q^2 + \|u(t+k)\|_R^2, \tag{7}$$

where, $R \succ 0$ and $Q \succeq 0$ are block diagonal weighting matrices of appropriate dimensions penalizing the control signal and the deviation of the agent's speed from the desired value, respectively. The formal centralized problem for traffic coordination at traffic intersections is given by

$$\begin{aligned} \min_{[u(t), u(t+1), \ldots]} \quad & J_{\mathrm{centr},t} \\ \text{subject to:} \quad & (1), (2) \text{ and } (3),\ \forall\ i \in \mathcal{N} \\ & (6),\ \forall\ i, j \in \mathcal{N}, j \neq i. \end{aligned} \tag{8}$$

The solutions to (8) consists of the best possible strategies with respect to (7), given the geometry of the coordination scenario, the initial state configuration and the limitations and capabilities of each vehicle. However, the structure of (8) raises serious computational complexity issues related to the prediction horizon length and the number of involved vehicles. Indeed, even a formulation of (8) over a finite time horizon W, yet large, might be computationally prohibitive. In order to tackle this issue, we propose in the sequel a low complexity decentralized solution based on a pre-defined decision order.

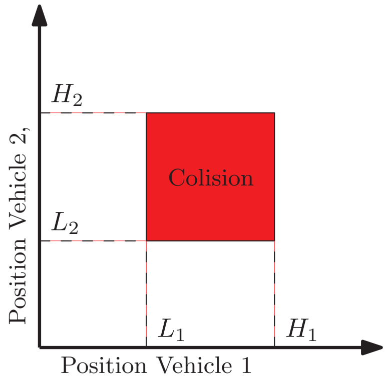


Fig. 2. Schematic illustration of a two vehicle collision accordingly to Definition 1 and (6). The red area contains the infeasible (forbidden) state space combinations and corresponds to collisions between vehicles 1 and 2.

4. DECENTRALIZED PROBLEM FORMULATION

Due to the collision avoidance constraints (6), problem (8) is computationally prohibitive and non-convex, see Figure (2). However, a low complexity approximate solution of problem (8), based on the sequential solution of $2N-1$ convex problems, can be formulated by assuming the existence of a decision order [Campos et al. (2013, 2014)]. In the context of this article, a decision order defines the sequence in which the different agents will solve their local optimization problems. The following definition is introduced.

Definition 3. (**Decision order**). Let $\mathcal{N} = \{1, \ldots, N\}$ be the set of vehicles and $\mathcal{O}$ a permutation of $\mathcal{N}$ defined according to a given criterium θ_i. Then $\mathcal{O}$ is considered to be the *decision order*, where $(\mathcal{O})_m$ denotes the m-th element in the order. Furthermore, $\mathcal{O}$ can be partitioned, with respect to each vehicle $i = (\mathcal{O})_m$, into $\mathcal{O}_i^b$ and $\mathcal{O}_i^a$: the first set contains the indices of all agents $j \neq i$ appearing before i in the decision order, while the second includes the indices of the vehicles appearing after agent i.

Considering a decision order, a sequential approach is proposed here. It is assumed that the first agent on the decision order will progress according to a local unconstrained optimization problem, which will broadcast the expected occupancy interval of the intersection. This will then be used by the remain vehicles on the decision order to enforce collision avoidance in their respective local optimization problems (problems (14) and (15) presented later in this paper).

4.1 Priority ordering

The main idea behind the proposed decentralized approach relies on a pre-defined decision order establishing in which sequence agents will solve their local optimization problems. How to define a meaningful decision order, however, is still an open problem. Here, we consider commonly used heuristics for priority assignment. We will also propose a novel criteria able to incorporate the individual degree of freedom of each vehicle.

(1) **First In First out (FIFO):** FIFO algorithms are methods for organizing and manipulating a queue of operations, widely spread in different technical fields. They can also be applied to intersection crossing where the vehicle arriving first to the intersection, or "head" of the queue, is processed first. In the context of this paper, vehicles getting to the intersection earlier will then gain priority in the decision sequence $\mathcal{O}$.

(2) **Distance to intersection:** Another intuitive, commonly used decision criteria is based on the distance between each vehicle and the collision point. In the context of this paper, such algorithm has the advantage of handling eminent collisions first, while keeping far-way agents at the end of queue. Nevertheless, such approach is incapable of incorporating the effective control freedom of each vehicle.

(3) **Time to react:** The concept behind this approach relies on the set of state configurations that will lead to an unavoidable collision. For example, it is possible that an agent, once it detects a possible collision with another agent, is unable to control its future trajectory in such a way that it can influence the time instants at which it will occupy the intersection. Based on such an argument, we are interested in determining if it exists a trajectory leading a vehicle to the critical set in a finite number of steps, under any feasible control input [Campos et al. (2013)]. The following definition, taken from [Borrelli et al. (2014)], will be used in the sequel to define the degree of freedom of each vehicle.

Definition 4. (**One-step robust controllable set**). Consider a system given by

$$x(t+1) = f(x(t), w(t)),$$

where $x(t) \in X, w(t) \in W,$ and $t \geq 0$. We denote the one-step controllable set to the set $\mathcal{T}$ as

$$\mathrm{Pre}(\mathcal{T}, \mathcal{W}) \triangleq \{x \in X \ : f(x,u) \in \mathcal{T}, \forall w \in \mathcal{W}\}.$$

In a intersection scenario, it is clear that the individual degree of freedom of a vehicle is defined by its range of feasible control inputs. Exploiting the notion of one-step robust controllable set, we introduce here the concept of *attraction set* denoted by $\mathcal{A}_i, \forall i \in N$. Using reachability analysis tools, the set $\mathcal{A}_i$ is defined as:

$$\begin{aligned} \mathcal{A}_i(\mathbf{Cr_i}) &= Pre(\mathbf{Cr_i}, U_i) \\ &= \{x_i(t) \ \in X_i : x_i(t+1) \in \mathbf{Cr_i}, \forall \ u_i \in U_i\}. \end{aligned} \tag{9}$$

In other words, the set $\mathcal{A}_i$ includes all possible state configurations that will lead the agent, unavoidably, to its *critical set* $\mathbf{Cr_i}$ in one step. The reader can refer to [Borrelli et al. (2014)] for further details on reachability/controllability analysis. In a general way, define now:

$$\begin{aligned} \mathcal{A}_i(\mathcal{T}) &= Pre(\mathcal{T}, U_i) \\ &= \{x_i(t) \ \in X_i : x_i(t+1) \in \mathcal{T}, \forall \ u_i \in U_i\}. \end{aligned}$$

where $\mathcal{T}$ is usually referred to as the target set. Denote $\mathcal{A}_{i1} = \mathcal{A}_i(\mathbf{Cr_i})$. By performing backward sequential calculations, it is possible to compute the super-set $\mathcal{A}_i^z$ including all the *attraction sets* that will drive vehicle i to $\mathbf{Cr_i}$ in at most z steps such that:

$$\mathcal{A}_i^z = [\mathcal{A}_{i1}, \mathcal{A}_{i2}, \cdots, \mathcal{A}_{iz}], \tag{10}$$

where $\mathcal{A}_{in} = \mathcal{A}_i(\ \mathcal{A}_{i(n-1)})$. Note that the scalar z is not a chosen parameter but it rather depends on the structure of the problem. In other words, z is the largest scalar before the backward reachability calculations result in an empty set. For the sake of clearness, an illustrative schema is presented in Figure 3.

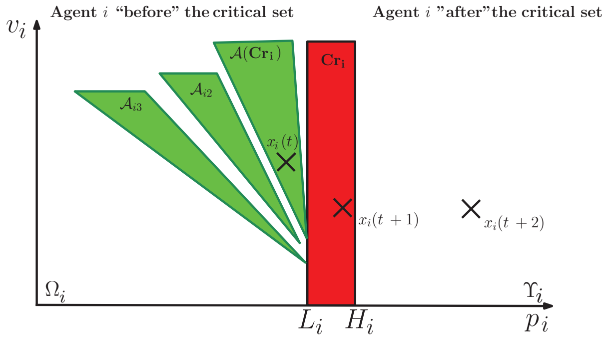


Fig. 3. Illustration of the attraction sets $\mathcal{A}_i^z$ and the critical set $\mathbf{Cr_i}$.

It follows from (4) and (10) that both $\mathbf{Cr}_i$ and $\mathcal{A}_i^z$ are time invariant, therefore offering precious information regarding the individual degree of freedom of each agent. We therefore propose to use such concepts to establish cooperation among agents. In order to introduce some logical fairness in the protocol and to compensate the natural drawbacks of a sequential decision procedure, in consider int this work a *control priority* defined in a proportionally inverse way with respect to the values of the "Time to react", defined as follows:

Definition 5. (**Time to react** Δ_i^{TR}). Consider the state $x_i(t)$ at the current time instant t. The time interval until the vehicle reaches any set n of $\mathcal{A}_i^z$ (if vehicle keeps its desired profile) is considered to be the **time to react** Δ_i^{TR}. By definition, it follows that the agent's will enter, unavoidably, into $\mathbf{Cr_i}$ in $\Delta_i^{TR} + n$ steps, $n \in \{1, \ldots, z\}$.

In other words, priority will be given to the agent lying closer to its attraction set, *i.e.*, the agent with the lowest Δ_i^{TR} value, then to the agent with the second smallest Δ_i^{TR} and so on. Such policy is motivated by the claim that the agent with the lower Δ_i^{TR} has, among all vehicles, the lowest individual degree of freedom.

4.2 Formulation of two convex optimization problems

Consider that the intersection is already occupied by an agent j during some time interval $\Gamma_{j,t}$. Thus, only two options are therefore valid for any other agent: (i) crossing the intersection before; (ii) crossing the intersection after. Extending this line of thought, a decentralized scheme can be set up where each agent in the decision order restricts its choices to pass the intersection before or after *all* the preceding vehicles in the decision order. Note that this approach is no longer optimal, since not all crossing orders are explored [1]. Note that the computational complexity of the proposed algorithm is linear with the vehicles number N, relying on the sequential computation of $2N-1$ quadratic programming problems. For the general case of N vehicles (with $N!$ potential crossing orders), the proposed approach reduces the number of possible crossing orders to $2^{(N-1)}$ and considers only one of them.

Keeping the same performance metrics as in (7), the local objective associate to each agent $i \in \mathcal{N}$ is given by

$$J_{i,t}^d = \sum_{k=0}^{W} \|v_i(t+k) - v_{di}\|_{Q_i}^2 + \|u_i(t+k)\|_{R_i}^2, \quad (11)$$

where W is a very large prediction horizon and $R_i \succ 0$ and $Q_i \succeq 0$ are weighting matrices of appropriate dimensions. We can informally define the following two problems:

- **Problem A (Informal Statement):** Find the optimal control policy such that agent i enters the intersection only after *all* preceding agent(s) $j \in \mathcal{O}_i^b$ have exited.

- **Problem B (Informal Statement):** Find the optimal control policy such that agent i exits the intersection before *any* preceding agent(s) $j \in \mathcal{O}_i^b$ enters.

For agent i, let $\Psi_{i,t} = \bigcup_{j\in\mathcal{O}_i^b} \Gamma_{j,t}$ be the union of the occupancy intervals of all preceding vehicles in $\mathcal{O}$. Collision avoidance is then ensured if

(1) For Problem A, the earliest entry time for agent i is given by
$$t_{i,t}^a = \max_{m\in\Psi_{i,t}} \{m\} + \delta_i^a. \quad (12)$$
(2) For Problem B, the latest exit time for agent i is given by
$$t_{i,t}^b = \min_{m\in\Psi_{i,t}} \{m\} - \delta_i^b. \quad (13)$$

Here $\delta_i^b,\ \delta_i^a \in \mathbb{Z}_+$ are safety time gaps between two occupancy intervals. We are now ready to formulate Problems A and B as two convex optimization problems where collision avoidance is enforced by state constraints. Thus, we have

Problem A:

$$\begin{aligned} \min_{[u_i(t),u_i(t+1),\ldots]} \quad & J_{i,t}^d \\ \text{subject to:} \quad & (1),\ (2) \text{ and } (3), \\ & p_i(t_{i,t}^a) \in \Omega_i. \end{aligned} \quad (14)$$

Problem B:

$$\begin{aligned} \min_{[u_i(t),u_i(t+1),\ldots]} \quad & J_{i,t}^d \\ \text{subject to:} \quad & (1),\ (2) \text{ and } (3), \\ & p_i(t_{i,t}^b) \in \Upsilon_i. \end{aligned} \quad (15)$$

Assuming that both Problem A and B have a solution, each vehicle will then choose the solution associated with the lowest $J_{i,t}^d$.

4.3 Feasibility analysis

From problems (14) and (15), one can easily conclude that the proposed conflict resolution algorithm relies on two optimization problems over two different horizons: one guaranteeing that a vehicle i can reach Υ_i in $(t_{i,t}^b - t)$ steps (i.e., accelerating and crossing first); the other ensuring that the agent can remain within Ω_i in $(t_{i,t}^a - t)$ steps (i.e., slow down and take the last position). The following definition is taken from [Borrelli et al. (2014)] and is used in the sequel to derive feasibility conditions of a given decision order.

Definition 6. (**One-step and R-step controllable sets**). Consider a system subject to external inputs given by
$$x(t+1) = f(x(t), u(t)),$$
where $x(t) \in X, u(t) \in U,$ and $t \geq 0$. We denote the one-step controllable set to the set $\mathcal{T}$ as
$$\text{Pre}(\mathcal{T}) \triangleq \{x \in X \ : \exists u \in U \text{ s.t. } f(x,u) \in \mathcal{T}\}.$$
Furthermore, the R-step controllable set $\text{K}^R(\mathcal{T})$ to the set $\mathcal{T}$ is defined recursively as
$$\text{K}^m(\mathcal{T}) \triangleq \text{Pre}(\text{K}^{m-1}(\mathcal{T})) \cap X, \quad \text{K}^0(\mathcal{T}) = \mathcal{T},$$
where $m \in \{1, \ldots, R\}$.

For a given decision order, the following conditions hold [Campos et al. (2014)].

Proposition 1. (**Local feasibility**). Let the state of an agent $i \in \mathcal{N}$, driven by dynamics (1), be $x_i(t) \in X_i$ at time t. Given a decision sequence $\mathcal{O}$, agent i has a feasible solution if and only if **at least one** of the following conditions is satisfied
$$x_i(t) \in \text{K}_i^{(t_{i,t}^a - t)}(\Omega_i), \quad (16a)$$
$$x_i(t) \in \text{K}_i^{(t_{i,t}^b - t)}(\Upsilon_i), \quad (16b)$$

It follows from Def. 6 that if condition (16a) is satisfied, then $\exists u_i \in U_i$ such that vehicle i can remain within Ω_i in $(t_{i,t}^a - t)$ steps. On the other hand, if condition (16b) is satisfied, then there exists a feasible control input that can drive the system to the target set Υ_i in $(t_{i,t}^b - t)$ steps. Thus, if one of these is satisfied, there exists at least one feasible control sequence satisfying the safety constraints.

Proposition 2. (**Global feasibility**). Consider a set of N systems driven by dynamics (1) such that $x(t) \in X$. At time t, a decision order $\mathcal{O}$ is feasible if and only if Proposition 1 is satisfied for each element in $\mathcal{O}$, except the first one.

Proposition 1 and Proposition 2 present local and global feasibility conditions for a given decision order $\mathcal{O}$, respectively. More precisely, they allow us to verify feasibility

[1] As an example, take the three vehicle case, with a decision sequence $\mathcal{O} = \{1, 2, 3\}$. Considering the proposed algorithm, only four possible crossing orders {1,2,3}, {3,1,2}, {2,1,3} and {3,2,1} are considered, whereas sequences {1,3,2} and {2,3,1} are discarded.

of an order by performing set-membership tests according to Proposition 1 and 2. From an implementation point of view, this can help to reduce the computational load considering that the derivation of the backward reachable sets can be locally pre-computed.

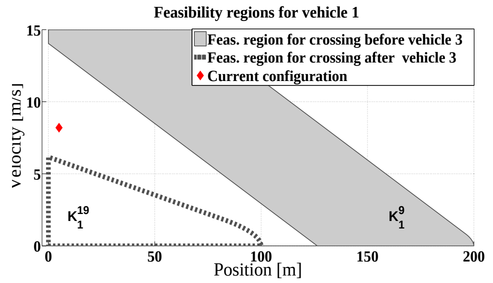
***Remark** 1.* Note that the concept of *decision sequence* has been extended in [Campos et al. (2014)] to a receding horizon control (RHC) framework. For any given decision sequence, it was shown that highly complex coordination scenarios can be simplified into simple and scalable RHC problems. More precisely, the local problems are divided into a finite-time optimal control problem, where collision avoidance is enforced as terminal constraints, and an infinite horizon control problem that is solved offline.

5. SIMULATION RESULTS AND DISCUSSION

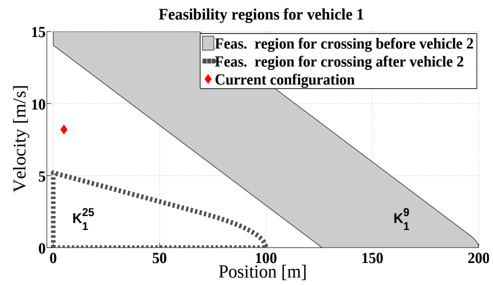
Consider a system of three vehicles ($N = 3$) as represented in Figure 1. Here, the safety parameter δ has been chosen as $\delta = [\ \delta^b\ \ \delta^a\ ]^T = [1\ \ 1]^T$ and $L_i = 100$ and $H_i = 130$, $\forall i \in \mathcal{N}$. Furthermore, vehicles are heterogeneous with respect to the control constraints such that $U_i \neq U_j$, $\forall i, j \in \mathcal{N}$. For each agent, the initial state is given by $x_i(0) = [p_i(0)\ v_{di}]^T$ such that $x_1(0) = [7\ 8.2]^T, x_2(0) = [4\ 5.95]^T, x_3(0) = [70\ 3.3]^T$. This yields $\Gamma_{1,t} = \{10 - 15\}, \Gamma_{2,t} = \{17 - 21\}$ and $\Gamma_{3,t} = \{10 - 18\}$. If no collision avoidance procedures are implemented, i.e., if all agents respect their pre-defined trajectory, then a collision can occur from $t = 10$ until $t = 21$.

In the sequel, we will discuss the advantages of the proposed sequential approach. As previously mentioned, there are several heuristics that can be used to define a decision order, as considered in the context of this paper. However, not all criteria θ_i offer the same performance and feasibility properties. In this work we are interested in the local and global feasibility of the decision order accordingly to Propositions 1 and 2. Note that the order $\mathcal{O}$ is assumed to be defined from the lowest to the highest value of $\theta_i, \forall i \in N$. Our goal is to show the advantages, in terms of feasibility, of a decision order based on time to react of the different vehicles, as proposed in this work. For comparison purposes, we consider other common decision criteria θ_i based on: (i) First In First Out (FIFO) protocols; (ii) distance to intersection.

For the different criteria, the resulting orders and feasibility arguments are presented in Table 1. Recall the intuition behind the concepts "time to react" and the attraction sets. Able to incorporate the individual degree of freedom of each vehicle, we claim here that such an approach leads to a deeper and comprehensive understanding of the current traffic situation, allowing therefore the derivation, from a fairness and feasibility point of view, of better priority relations/orders. The analysis of Table 1 supports our claims in terms of feasibility. Indeed, if a decision order is based on the "time to react", this leads to a feasible solution allowing agents to cross safely the intersection. On the other hand, however, the remaining criteria do not guarantee global feasibility, since both problems (14) and (15) are unfeasible for vehicle 1. This is shown in Figure 4, where the results of set-membership conditions



(a)



(b)

Fig. 4. Feasibility regions for vehicle 1: (a) for an order $\mathcal{O} = \{3, 1, 2\}$ defined with respect to a FIFO algorithm; (b) for an order $\mathcal{O} = \{3, 2, 1\}$ defined with respect to approaching speed $\|p_i(0) - L_i\|$. In both figures, the current state (red dot) does not belong to any of the presented sets, therefore compromising the global feasibility of the corresponding order.

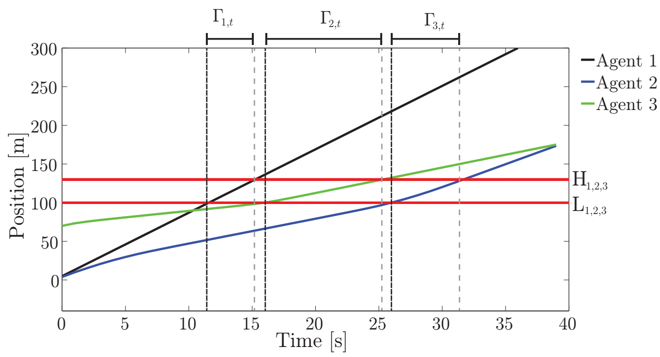


Fig. 5. Evolution of the agents' trajectories according to proposed sequential algorithm. The intersection is represented by the horizontal red lines and the black and grey dashed lines represent the entrance and exit time instants, respectively.

(accordingly to Proposition 1 and 2) are presented. In both figures one can see that the current configuration of vehicle 1 does not belong to any of the sets (derived using reachability theory). Accordingly to these results, vehicle 1 is: **(i)** unable to cross before vehicle 3 (in Fig. 4(a) and (b), $x(t) \notin K_1^9$); **(ii)** unable to cross after vehicle 3 (in Fig. 4(a), $x(t) \notin K_1^{19}$); **(iii)** unable to cross the intersection after vehicle 3 and 2 (in Fig. 4(b), $x(t) \notin K_1^{25}$).

Finally, Figure 5 presents the trajectories of the different vehicles according to the proposed control protocol for a decision order $\mathcal{O} = \{1, 3, 2\}$. Here, the critical set Cr_i is represented by the horizontal red lines and the black and grey dashed lines represent the entrance and exit times,

θ_i	"time to react"	$\|p_i(t) - L_i\|$	FIFO
Order $\mathcal{O}$	$\{1,3,2\}$	$\{3,2,1\}$	$\{3,1,2\}$
Feasibility	**Feasible**	**Unfeasible**	**Unfeasible**

Table 1. Decision orders $\mathcal{O}$ and feasibility conclusions with respect to different θ_i.

defining $\Gamma_{i,t}, \forall i \in \mathcal{N}$. One can observe that a collision is avoided, since the different $\Gamma_{i,t}$ never intersect. We recall that in the proposed approach time-dependent state constraints are enforcing collision avoidance conditions, see (14) and (15). According to the proposed algorithm, agent 1 (with highest priority) keeps its desired trajectory, crossing the intersection during $12 < t < 15$. Based on this information, agent 3 solves problems (14) and (15), choosing the best local solution. The same procedure is later performed by agent 2.

6. CONCLUSIONS

In this paper, we presented a cooperative conflict resolution approach for traffic intersections, based on a sequential approach. The proposed solution offers several advantages such as low complexity and scalability. In fact, its per agent complexity with respect to the number of agents remains constant since collision avoidance is enforced through local state constraints at given time steps. Furthermore, due to its low computational requirements, the proposed structure can also be cast into a RCH framework. Finally, this novel control formulation has also the merit of allowing the derivation of (easily verifiable) feasibility conditions for a given sequence. Throughout simulation results, we showed that a decision order based on the time to react, as proposed in this paper, guarantees global feasibility of the order whenever other decision criteria do not. To complement these results, future research should focus on the optimality analysis and experimental implementation of such approaches.

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