Distributed Labeling in Autonomous Agent Populations*

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1 Introduction

The idea of deploying formations of relatively unsophisticated autonomous agents to accomplish complicated tasks has roots in the early works studying the flocking and foraging behaviours among birds. The question of interest then was how one can mimic different behaviours witnessed in populations of birds, animals, insects, etc. among a population of artificially constructed agents. In 1987 the behaviour of a flock of birds (boids) in motion was modeled and simulated by Reynolds. The model relied on assuming local motion strategies for each of the boids; i.e. only local information from neighbouring boids was used by each individual. Many other works in coming years followed suit and contributed to our understanding of flocking like behaviours.

However, something that has not gained much attention is that in an ever changing dynamical environment the agents need to be able to communicate with each other to improve their adaptability and the overall performance of the group. In the animal kingdom this may be done via the secretion of pheromones or, in the particular example of humans, it is done by communicating via Language[1].

This work is an endeavour to study the possibility of artificial agents developing their own Language in the context of exploring their environment. We assume that there is no global frame of reference and there are certain prominent environmental features with unknown global positions, *beacons or landmarks*, such as a communication antenna or an urban landmark, that all the agents can recognise and can assign a label to. In addition, we assume that these labels are determined locally by each of the agents.

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[1] The capital L is used to highlight that we are discussing the idea of language not the examples of languages in use, as in not a language in all its specifics. We mean language in its French sense of *Langue* and not *parole*.

We are interested in investigating the possibility of agents determining each others positions via communicating their own position relative to these locally chosen labeled beacons. To achieve this, it is of course necessary for the agents to agree on the same name for the landmarks, and in a sense develop a Language of their own.

We formalize the problem of interest and outline a sketch solution in the remainder of this abstract.

2 Problem Formulation

Consider a set of n mobile agents indexed in $\mathcal{X} \triangleq \{i\}_{i=1}^n$ with global positions $\mathbf{x}_i(t) \in \mathbb{R}^2$ at time $t \in \mathbb{Z}_{\geq 0}$ (note that the agents do not know their global positions (or the global frame) as they have only access only to their own local frame). Agent i can communicate with another agent j at time t if $\delta_{ij}(t) \triangleq \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| \leq r_d$, where r_d, the communication radius, is a positive constant. Let $\mathcal{N}_i \triangleq \{k \mid \delta_{ik}(t) \leq r_d\}$ be the neighbor set of agent i; i.e. agent i can communicate with all $j \in \mathcal{N}_i$. Consider also a set of m environmental features (beacons) $\mathcal{B} = \{b_j\}_{j=1}^m$, where b_j is the global label of beacon j unknown to the agents. Each beacon j has a global position $\mathbf{b}_j \in \mathbb{R}^2$ unknown to the agents. The workspace is then divided into several cells such that each beacon is located in precisely one cell (i.e. no more or no less). We use $\mathcal{C}_j$ to denote the cell associated with b_j. Agent i identifies cell $\mathcal{C}_j$ by the label b_{ij}. We actually assume that each individual agent i labels each of the beacons $j \in \mathcal{B}$ locally, i.e. $\mathcal{L}_i \triangleq \{b_{ij}\}_{j=1}^m$, via a language function (bijection) $\ell_i : (\mathcal{B}, \mathbb{Z}_{\geq 0}) \mapsto \mathcal{L}_i$, i.e. $\ell_i(b_j, t) = b_{ij}$. For this paper, assume the cells are *Voronoi* cells and the beacons are the generators.

ASSUMPTION 1. *It holds that $\ell_i(b_j, 0) \neq \ell_k(b_j, 0)$, $\forall j \in \{1, \ldots, m\}$ and b_j and $b_{ij}(0)$ are uniformly distributed between 0 and $\bar{b} \in \mathbb{R}$. The agents are not equipped with relative position sensing; i.e. agent i has no access to the position of agent $j \neq i$. Each agent measures the range to those generators satisfying $r_{ij}(t) = \|\mathbf{x}_i(t) - \mathbf{b}_j(t)\| \leq r_d$.*

Suppose the agents are tasked with exploring the environment according to some motion strategy and in doing so they come into contact with each other and share their position in terms of their local cell identity; e.g. b_{ij}. The main problem that we consider in this work can then be stated at a high-level as follows.

PROBLEM 1. *Characterize the requirements on the agent's motion such that as $t \to \infty$ it follows that $\ell_i(b_j, t) = \ell_k(b_j, t)$, $\forall i, k$ and for all $j \in \{1, \ldots, m\}$ and $\nexists (j, l)$ with $j \neq l$ such that $\ell_i(b_j, t) = \ell_i(b_l, t)$. That is, characterize the requirements on the motion of each agent i such that as $t \to \infty$ the label for each beacon converges to a common value among all the agents (and no two beacons share the same value).*

The consequence (and significance) of achieving $\ell_i(b_j, t) = \ell_k(b_j, t)$, $\forall i, k$ and $\forall j \in \{1, \ldots, m\}$ is that subsequently each agent can transmit to their neighbours the identity of the cell they are in and their neighbours will understand the spatial meaning of this label.

The assumption that $\ell_i(b_j, 0) \neq \ell_k(b_j, 0)$ is motivated by the fact that in several applications different types/generations of robots might be present or in many cases there is no need, desire or ability to pre-program the agents with common beacon labels.

We outline an algorithm to address Problem 1 and follow this by providing a theorem which characterises the conditions on the agent's motion such that convergence occurs.

ALGORITHM 2.1 Exploration and Language Evolution at Agent i

$T_0 \leftarrow 0$;
for $l \in \mathbb{N}$ **do**
  $\mathcal{E}_i^j(l+1) \leftarrow \emptyset$;
  for $t \in [T_l, T_{l+1})$ **do**
    if $\mathcal{N}_i = \emptyset$ **then**
      Move agent i according to the motion strategy $\mathcal{S}_i$
    else
      Determine $\mathcal{C}_j$ such that $\mathbf{x}_i(t) \in \mathcal{C}_j$ by solving $j = \text{argmin}_j\, r_{ij}(t)$
      while $r_{ij} \geq \epsilon$ for some small $\epsilon > 0$ **do**
        Move towards $\mathbf{b}_j$
      end while
      if $\mathcal{N}_i \neq \emptyset$ **then**
        Send b_{ij} to all $k \in \mathcal{N}_i^b \triangleq \{k \mid \|\mathbf{x}_i - \mathbf{x}_k\| \leq r_i^b(t)\}$
        $\mathcal{E}_i^j(l+1) \leftarrow \mathcal{E}_i^j(l+1) \cup (i,k), \forall k \in \mathcal{N}_i^b$
        Receive b_{kj} from all $k \in \mathcal{N}_i^r \triangleq \{k | i \in \mathcal{N}_k^b\}$
        $\mathcal{E}_i^j(l+1) \leftarrow \mathcal{E}_i^j(l+1) \cup (k,i), \forall k \in \mathcal{N}_i^r$
        Set $b_{ij} \leftarrow \frac{1}{|\mathcal{N}_i^r|+1}[s_i b_{ij} + \sum\limits_{k \in \mathcal{N}_i^r} b_{kj}]$
      end if
    end if
  end for
  $\mathcal{E}^j(l+1) \leftarrow \cup_i \mathcal{E}_i^j(l+1), \forall j \in \{1, \ldots, m\}$
end for

Note that particular $\mathcal{S}_i,\ \forall i$ will be discussed in an extended version of this work but in the following theorem we give a sufficient condition on those $\mathcal{S}_i$ that ensure convergence. The distance $r_i^b(t)$ must satisfy $r_i^b(t) < r_d - \epsilon$. Finally, $s_i \neq s_j > 0$ is a selfishness factor.

THEOREM 1. *Consider the scenario described above with Assumption 1 and Algorithm 2. Define $\mathcal{G}^j(l+1)$ as the graph defined by the vertex set $\mathcal{X}$ with edge set $\mathcal{E}^j(l+1)$. Suppose there exists an $\mathcal{S}_i$, $\forall i$ such that there exists an infinite sequence of contiguous, non-empty and bounded time intervals $[T_l, T_{l+1})$ such that $\mathcal{G}^j(l+1)$ for all $l \in \mathbb{N}$ and for all $j \in \{1, \ldots, m\}$ is strongly connected. Then as $t \to \infty$ for all pairs (i,k) it follows that $\ell_i(b_j,t) = \ell_k(b_j,t)$ for all $j \in \{1, \ldots, m\}$ and $\nexists(j,l)$ with $j \neq l$ such that $\ell_i(b_j,t) = \ell_i(b_l,t)$.*