Communication Analysis for Centralized Intersection Crossing Coordination

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***Abstract*—Coordination of autonomous cooperative vehicles is an important challenge for future intelligent transportation systems. In particular, coordination to cross intersections captures the inherent and connected challenges among control and communication. While intersection coordination and vehicular wireless communication have both received extensive treatment in their respective communities, few works consider their interaction. We provide a communication system analysis for the specific problem of centralized intersection crossing coordination, leading to design guidelines for both uplink (whereby vehicles send intentions to the central controller) and downlink (where the controller prescribes vehicles of safe control actions).**

I. Introduction

Road intersections are among the most complex and accident-prone elements of modern traffic systems, accounting for 43% of the total injury causing accidents and 21% of the vehicle related fatalities in the EU [1]. Consequently, intersections are also among the most controlled traffic situations, often regulated simultaneously by right-of-way rules, signs, and traffic lights. The complexity and comprehensive regulation means that intersections often form bottlenecks in the traffic system, where the average speed and traffic throughput drops significantly.

In order to improve the efficiency of the road infrastructure and to achieve safer operation, several studies have considered cooperative strategies for intersection scenarios. In these scenarios, vehicles share information that is used to make a global decision and to reach agreement [2]–[9]. This process can be either centralized or distributed. More specifically, [2], [3] present a supervisory controller that overrides the driver's commands if these take the vehicle out of the maximum control invariant set with respect to a collision inside the intersection. It is shown that this control problem is NP-hard, but that a fully polynomial time approximation scheme exists. This intervention-based approach is elaborated in [4], [5], where experimental results are provided. In the context of fully autonomous vehicles, several studies have been made on both decentralized strategies and on those that to some degree rely on a central computational unit. Decentralized approaches based on predictive control and reachability analysis [6], [7] and on navigation function controllers [10] have been considered. A somewhat different approach is taken in [11], where the authors abstract from the dynamics and present a number of protocols for distributed decision making in the intersection context. A partly centralized, and provably safe strategy is presented in [9] based on centralized scheduling and subsequent time-slot assignment for individual vehicles. Similar ideas are put forward in [12].

In the above works, decisions are made under the assumption of perfect information exchange between the involved vehicles and/or central infrastructure, meaning that the problems inherent to wireless communication (e.g., packet losses and delays) are largely ignored. Research on vehicular communication, on the other hand, has mainly focused on measurement campaigns and simulations for vehicular channel characterization without considering specific applications [13], [14]. However, it is clear that different applications within the automotive domain have different requirements on the communication links and that safety-related applications considered in this paper are among the ones imposing the most stringent demands. In order to avoid extensive simulations and measurements to validate a communication system design, recent work has focused on the derivation of analytical expressions of key performance metrics for different scenarios utilizing stochastic geometry [15]. For example, [16], [17], consider Poisson and Cox processes for modeling of clustering, while [18] presents a reception probability model for vehicular ad-hoc networks (VANETs) in the vicinity of intersections. However, these works do not consider specific safety-related traffic applications.

In this paper, we aim to reduce the gap between the control algorithms and the underlying communication system. We present a framework for analyzing the performance of the communication system in an intersection scenario, where the coordination and conflict resolution is handled by a centralized computational unit (controller) whose commands are followed by the individual vehicles. The specifications on the controllers are kept general within this context in order to allow easy integration with both existing and future work on the control aspects of the problem.

II. System Model

System Entities: We consider a two-road intersection, with one incoming lane per road, as shown in Fig. 1. On each

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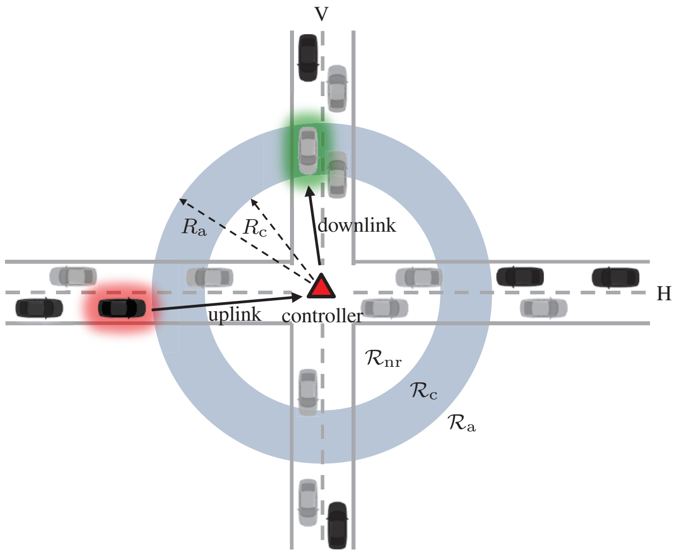

Figure 1. System model consisting of an intersection scenario with two perpendicular roads. The controller which is located in the center of the intersection receives state information (uplink) from a vehicle located in region $\mathcal{R}_a$ and broadcasts coordination information (downlink) to all vehicles in $\mathcal{R}_c$. The black vehicles in the region $\mathcal{R}_a$ all try to transmit state information to the controller and could potentially interfere with each other if transmitting in the same time slot.

incoming lane, there is an infinite stream of vehicles, characterized by a Poisson point process (PPP) with density λ, expressing the number of vehicles per meter of road traveling with a target velocity v, expressed in meters per second. For simplicity these values are equal for all roads. There is a central infrastructure unit located at the center of the intersection, which coordinates the crossing procedure. Vehicles transmit their state information to the controller with an update period T, i.e., time is discretized in slots, and in i slots, vehicles move viT meters. The position of an incoming vehicle on the horizontal road is denoted by $\mathbf{x} = [x, 0]^{\mathrm{T}}$, where for $x > 0$ the vehicles are incoming from the right, while for $x < 0$ the vehicles are incoming from the left. For notational convenience, we do not account for the size of the vehicles or distances among parallel lanes. Similarly, the position of an incoming vehicle on the vertical road is denoted by $\mathbf{x} = [0, y]^{\mathrm{T}}$.

Controller Operation: The controller operates as follows. We distinguish regions $\mathcal{R}_{\mathrm{nr}}$, $\mathcal{R}_{\mathrm{c}}$, and $\mathcal{R}_{\mathrm{a}}$ around the intersection (see Fig. 1), defining distances $R_{\mathrm{c}} < R_{\mathrm{a}}$. Vehicles in $\mathcal{R}_{\mathrm{a}}$ (i.e., at a distance greater than R_{a}) access the wireless channel to send their state information (position, velocity, destination direction) to the controller. When vehicles enter $\mathcal{R}_{\mathrm{c}}$ (i.e., at distances between R_{c} and R_{a}), they listen to the controller to learn how and/or when they are allowed to cross the intersection, and to which vehicles they must give way. Note that vehicles are assumed to move at their desired velocity v when in $\mathcal{R}_{\mathrm{c}} \cup \mathcal{R}_{\mathrm{a}}$. Once in $\mathcal{R}_{\mathrm{nr}}$ (i.e., at distances below R_{c}), the region of no return, the communication from the controller should be completed and the vehicle acts according to the received instructions.

Wireless Communications: Consider a vehicle located a distance d away from the center of the intersection. The channel between the vehicle and the controller is governed by Rayleigh multipath fading with exponential power fading $S \sim \exp(1)$ and path loss $l(d) = (Ad)^{-\alpha}$, where A captures antenna gains and α is the path loss exponent. At the receiver, the signal is further affected by additive white Gaussian noise with power σ^2. The communication relies on frequency division duplexing (FDD), so that uplink (where vehicles send information to the controller) and downlink (where the controller broadcasts coordination information) communications are not interfering. During uplink, vehicles in $\mathcal{R}_a$ send their state using a transmit power P_{ul} on one of N_{ul} available channels. This means that the location of the vehicles contributing to the interference in a certain time slot can be represented by two one-dimensional non-homogeneous PPPs, one for the horizontal road and one for the vertical road, denoted Φ_{H} and Φ_{V}, respectively. The density for both of these PPPs are dependent on the distance from the intersection and can be expressed as

$$\Lambda(d) = \begin{cases} \lambda/N_{\mathrm{ul}} & d \geq R_{\mathrm{a}}, \\ 0 & d < R_{\mathrm{a}}, \end{cases} \tag{1}$$

i.e., there are no interferers within the region $\mathcal{R}_{\mathrm{c}} \cup \mathcal{R}_{\mathrm{nr}}$. During the downlink, vehicles in $\mathcal{R}_{\mathrm{c}}$ listen to the controller, which broadcasts control information for all vehicles with a power P_{dl}. A packet in either uplink or downlink is received successfully when the signal-to-interference-plus-noise ratio (SINR) is above a certain threshold β. Note that the downlink is not subject to any interference as the only transmission on the downlink frequency is a broadcast from the controller.

Objective: Given the above scenario, our aim is to set the communication parameters (T, N_{ul} and P_{ul} in the uplink as well as P_{dl} in the the downlink) to ensure that each vehicle can be guaranteed with high probability $1-\varepsilon$ (say, 99%) that its state information is received by the controller when in $\mathcal{R}_{\mathrm{a}}$ and its respective control signal is obtained when in $\mathcal{R}_{\mathrm{c}}$.

III. System Analysis

A. Uplink Communication

We focus on a given vehicle, referred to as the ego vehicle $\mathbf{u}$, and determine how to satisfy the performance requirements. Its distance to the controller d reduces from $+\infty$ to R_{a} in steps of Tv. In order to achieve the target quality of service, the probability that all transmissions[1] fail, denoted $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T)$, should be smaller than ε:

$$\underbrace{\mathbb{E}_{\Phi_{\mathrm{H}},\Phi_{\mathrm{V}}}\left[\Pr\left\{\bigcap_{i=0}^{+\infty} \mathrm{SINR}(R_a + ivT) < \beta | \Phi_{\mathrm{H}}, \Phi_{\mathrm{V}}\right\}\right]}_{=\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T)} < \varepsilon. \tag{2}$$

We introduce $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(d|\Phi_{\mathrm{H}}, \Phi_{\mathrm{V}}) = \Pr\{\mathrm{SINR}(d) < \beta | \Phi_{\mathrm{H}}, \Phi_{\mathrm{V}}\}$ as the conditional failure probability, i.e., the probability that the controller is not able to decode a transmission from a vehicle located a distance d away given the location of the interferers. Note that the locations of the interferers change as the vehicle moves closer to the intersection, and that the

[1] Note that we assume that the last transmission occurs at $d = R_{\mathrm{a}}$, which leads to somewhat optimistic results.

successive failure events are correlated. This makes a closed-form evaluation of (2) difficult. Nevertheless, we can compute the probability that a given transmission fails, or that a short sequence of transmissions fails. We will focus on the case $\alpha = 2$, which corresponds to free-space propagation.

Proposition 1. *The probability that a transmission fails when the transmitter is at a distance d away from the intersection is given by*

$$\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(d) = 1- \exp\left(\frac{-\beta\sigma^2}{P_{\mathrm{ul}}l(d)}\right)\exp\left(-4d\frac{\lambda}{N_{\mathrm{ul}}}\sqrt{\beta}\mathrm{arccot}\left(\frac{R_{\mathrm{a}}}{\sqrt{\beta}d}\right)\right). \quad (3)$$

Proof: See Appendix A. ■

Proposition 2. *The probability that two successive transmissions (the first at distance d, the second at distance $d - vT \geq R_{\mathrm{a}}$) fail is given by*

$$\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(d, d-vT) = 1 - \mathbb{P}_{\mathrm{ul}}^{\mathrm{succ}}(d) - \mathbb{P}_{\mathrm{ul}}^{\mathrm{succ}}(d-vT) + \exp\left(-\frac{\beta\sigma^2}{P_{\mathrm{ul}}}\left(\frac{1}{l(d)}+\frac{1}{l(d-vT)}\right) - 2\frac{\lambda}{N_{\mathrm{ul}}}\int_0^{+\infty}(1-g(x))\,\mathrm{d}x\right) \quad (4)$$

in which $g(x)$ is defined in (42).

Proof: See Appendix B. ■

These results can now be used to find bounds on the failure probability (2).

1) Lower bounds: A simple lower bound is given by Harris' inequality [19]

$$\begin{aligned}
&\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T) \\
&\geq \prod_{i=0}^{+\infty} \mathbb{E}_{\Phi_{\mathrm{H}},\Phi_{\mathrm{V}}}\left[\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}} + ivT|\Phi_{\mathrm{H}},\Phi_{\mathrm{V}})\right] \quad (5)\\
&= \prod_{i=0}^{+\infty} \mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}} + ivT), \quad (6)
\end{aligned}$$

which can be computed using Proposition 1. A tighter lower bound, again using Harris' inequality, can be found by considering pairs of successive transmissions:

$$\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T) = \Pr\left\{\bigcap_{i=0}^{+M}\mathrm{SINR}(R_a + ivT) < \beta\right\}, \quad (7)$$

where M is a large integer. These events are all correlated. Now,

$$\begin{aligned}
&\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T) \\
&\geq \Pr\{\mathrm{SINR}(R_a) < \beta\} \quad (8)\\
&\prod_{i=1}^{M}\Pr\{\mathrm{SINR}(R_a + ivT) < \beta | \mathrm{SINR}(R_a + (i-1)vT) < \beta\} \\
&= \mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}}) \prod_{i=1}^{M}\frac{\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}} + ivT, R_{\mathrm{a}} + (i-1)vT)}{\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}} + (i-1)vT)}, \quad (9)
\end{aligned}$$

which can be computed from Propositions 1–2. Tighter lower bounds can be obtained by considered interactions over more than two time slots, but at a cost in computational complexity, since more terms will appear in expressions such as (4), and each term will require numerical integration.

2) Upper bound: We can upper bound the failure probability by considering only the last two transmissions

$$\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T) \leq \mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(R_{\mathrm{a}} + vT, R_{\mathrm{a}}), \quad (10)$$

which can be evaluated using Proposition 2.

B. Downlink Communication

In the downlink, vehicles in $\mathcal{R}_c$ listen to the controller. In order to achieve the target quality of service, the probability that all transmissions[2] fails should be smaller than ε:

$$\underbrace{\prod_{i=0}^{\lfloor \frac{R_{\mathrm{a}}-R_{\mathrm{c}}}{vT}\rfloor}(1 - \mathbb{P}_{\mathrm{dl}}^{\mathrm{succ}}(R_{\mathrm{c}} + ivT))}_{=\mathbb{P}_{\mathrm{dl}}^{\mathrm{fail\_tot}}(P_{\mathrm{dl}},T)} < \varepsilon. \quad (11)$$

where $\mathbb{P}_{\mathrm{dl}}^{\mathrm{succ}}(d)$ is the probability that a vehicle at a distance d away from the controller successfully decodes a packet. As the downlink is not subject to any interference this success probability is simply

$$\begin{aligned}
\mathbb{P}_{\mathrm{dl}}^{\mathrm{succ}}(d) &= \Pr\{\mathrm{SINR}(d) > \beta\} \quad (12)\\
&= \Pr\left\{\frac{P_{\mathrm{dl}}Sl(d)}{\sigma^2} > \beta\right\} \quad (13)\\
&= \Pr\left\{S > \frac{\beta\sigma^2}{P_{\mathrm{dl}}l(d)}\right\} \quad (14)\\
&= \exp\left(\frac{-\beta\sigma^2}{P_{\mathrm{dl}}l(d)}\right), \quad (15)
\end{aligned}$$

and for the special case of $\alpha = 2$, we finally have

$$\mathbb{P}_{\mathrm{dl}}^{\mathrm{fail\_tot}}(P_{\mathrm{dl}}, T) = \prod_{i=0}^{\lfloor \frac{R_{\mathrm{a}}-R_{\mathrm{c}}}{vT}\rfloor}\left(1 - \exp\left(\frac{-\beta\sigma^2A^2(R_{\mathrm{c}} + ivT)^2}{P_{\mathrm{dl}}}\right)\right). \quad (16)$$

C. Overall Analysis

For the downlink, we aim to determine values for $[P_{\mathrm{dl}}, T]$ to ensure that $\mathbb{P}_{\mathrm{dl}}^{\mathrm{fail\_tot}}(P_{\mathrm{dl}}, T) < \varepsilon$. In general, there may be multiple downlink parameter values within this feasible set. Hence, we can aim to minimize the transmit power P_{dl} or the transmit power divided by the update period P_{dl}/T, where the latter objective is proportional to the total amount of energy spent. Inspection of (16) indicates that $\mathbb{P}_{\mathrm{dl}}^{\mathrm{fail\_tot}}(P_{\mathrm{dl}}, T)$ can be reduced by (i) increasing P_{dl}; (ii) reducing T to provide more transmission opportunities.

Similarly, for the uplink, we aim to determine good values (in terms of transmit power P_{ul}, transmit power divided by the update period P_{ul}/T, and the number of uplink channels N_{ul}) of $[N_{\mathrm{ul}}, P_{\mathrm{ul}}, T]$ to ensure that $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T) < \varepsilon$. Inspection of (3) and (5) indicates that to reduce $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T)$, we need to make both factors in (3) small. To reduce the first factor, we can increase P_{ul} or reduce T. To reduce the second factor, we can allocate more channels N_{ul} to reduce the interference per channel.

[2]Note that we assume that the last transmission occurs at $d = R_{\mathrm{c}}$, which again leads to somewhat optimistic results.

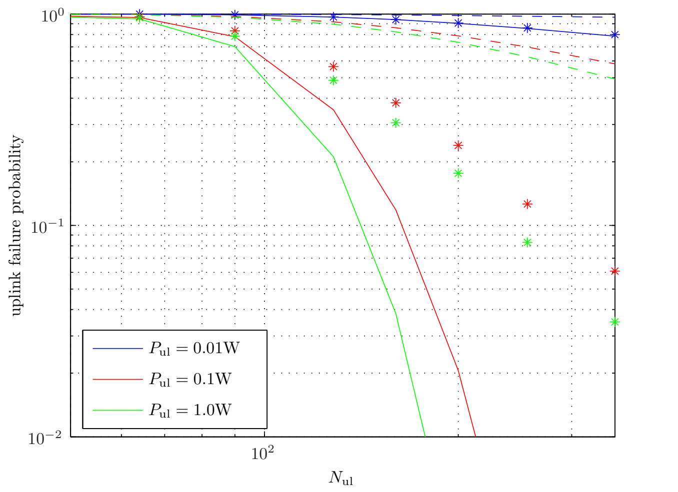

Figure 2. Uplink failure probability as a function of N_{ul}, the number of uplink channels, for varying uplink powers, and $T = 0.1\,\mathrm{s}$. Full lines corresponds to the lower bound (9), dashed lines to the upper bound (10), and the stars in the same color correspond to the numerical evaluation of $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T)$.

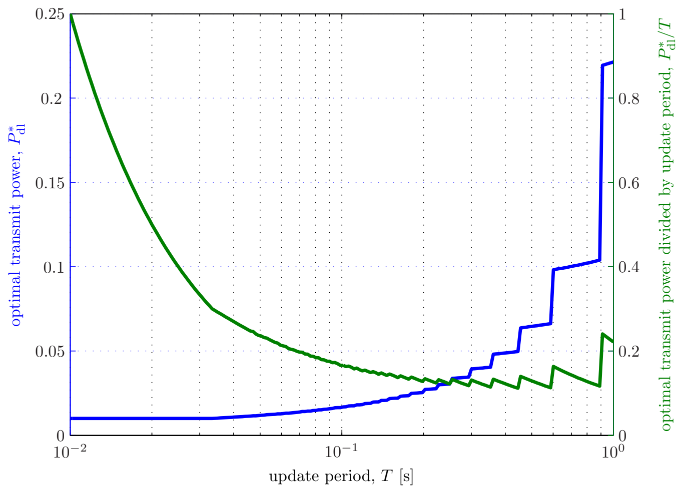

Figure 3. Minimum transmit power P_{dl}^* and transmit power divided by the update period P_{dl}^*/T for the downlink as a function of the update period T, under the constraint that $\mathbb{P}_{\mathrm{dl}}^{\mathrm{fail\_tot}}(P_{\mathrm{dl}}^*, T) < 0.01$.

IV. Numerical Results

A. Scenario

We have set $\lambda = 0.1$ (i.e., the average inter-vehicle spacing in each lane is 10 m), and $v = 100\,\mathrm{km/h}$ until entering the region $\mathcal{R}_{\mathrm{nr}}$. Assuming that vehicles obey to double integrator dynamics with a maximum deceleration of $2\,\mathrm{m/s^2}$, we set $R_{\mathrm{a}} = 300\,\mathrm{m}$, $R_{\mathrm{c}} = 250\,\mathrm{m}$. This means that each vehicle has approximately 10 seconds in $\mathcal{R}_{\mathrm{nr}}$, and that they all have, in a worst case scenario, the possibility of coming to a complete stop before the intersection. Furthermore, the chosen values on R_{a} and R_{c} implies that an average of 5 vehicles per road are in downlink. For the communication, we assume a noise variance σ^2 of $-99\,\mathrm{dBm}$, $A = 650$, a path-loss exponent $\alpha = 2$, and an SINR threshold of $\beta = 8\,\mathrm{dB}$ [14].

B. Results and Discussion

1) Uplink: We will set $T = 0.1\,\mathrm{s}$, which is a reasonable value according to VANET communication standards [14]. In Fig. 2, we show the uplink failure probability $\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail\_tot}}(N_{\mathrm{ul}}, P_{\mathrm{ul}}, T)$, obtained through Monte Carlo estimation over 200 PPP realizations, along with upper bound (10) and the lower bound (9). According to the lower bound, we observe that we need at least 100 channels and a relatively high uplink power to provide a reasonable quality of service, e.g., $\varepsilon = 0.01$. However, for low values of the failure probability the bounds are extremely loose and in reality we might require well in excess of 100 channels.

2) Downlink: In the downlink, we will vary T in the interval $[0.01\,1]\,\mathrm{s}$. For each value of T, if (11) has a feasible solution, there exists a smallest feasible power, say P_{ul}^* that guarantees (11). In Fig. 3 we visualize this minimal power as well as the optimal transmit power divided by the update period P_{ul}^*/T, as a function of T for $\varepsilon = 0.01$. In terms of transmit power, we see that very short update periods are preferred with very low transmit power. However, by studying the objective P_{ul}^*/T, we can see that the overall energy consumption will be large for short update periods. An optimal trade-off is $T \approx 0.6\,\mathrm{s}$, leading to the minimal value of the objective P_{ul}^*/T. Note that, due to the jagged nature of the curves, this is not a robust choice, so a more appropriate value would be around 0.3–0.4 s. This jagged behavior is due to the small number of transmission attempts in the region $\mathcal{R}_{\mathrm{c}}$, so that small increases in T can lead to an additional transmission attempt, thus a lower overall failure probability.

C. Impact on Control Algorithms

The communication systems analysis can be an important tool to complement the study of the controller's performance/optimality given certain communication parameters, quality of service requirements, or traffic density. On the other hand, it is important, for the sake of success of the negotiation procedure, that an agreement is reached before collisions become unavoidable. The proposed three communication regions setup could therefore be a useful tool to identify such "points of no return". By guaranteeing that reliable information is available before vehicles reach a certain distance from the intersection, alternative solutions such as emergency interventions should therefore also be considered. Finally, through a combined communication/control approach, one can come up with alternative formulations of the centralized negotiation problem, which could lead to an elegant integrated analysis of the underlying coordination problem.

V. Conclusions

We have provided a communication systems analysis for a centralized intersection crossing coordination scheme. In particular, we considered an FDD system with orthogonal channels on the uplink, wherein the wireless channel was modeled to comprise path loss and small-scale fading. Using techniques from stochastic geometry, we were able to derive system parameters for both uplink and downlink that are able to meet pre-described performance guarantees and simultaneously minimize the utilization of system resources such as power. Furthermore, we expect the tools developed in this paper to be of use for the control theory community, for example to study how far away from the intersection the controller can expect to have information available from all vehicles given certain communication parameters, quality of service requirements, vehicle densities, and velocities.

In future work we will derive tighter bounds for the uplink failure probability. In addition, we will perform an integrated analysis with the coordination algorithm.

Appendix A
Proof of proposition 1

The probability of a successful transmission, denoted by $\mathbb{P}_{\text{ul}}^{\text{succ}}(d) = 1 - \mathbb{P}_{\text{ul}}^{\text{fail}}(d)$, is defined as

$$\mathbb{P}_{\text{ul}}^{\text{succ}}(d) = \Pr\{\text{SINR}(d) > \beta\} \quad (17)$$

$$= \Pr\left\{\frac{P_{\text{ul}} S l(d)}{I_{\text{H}} + I_{\text{V}} + \sigma^2} > \beta\right\}, \quad (18)$$

where I_{H} (resp. I_{V}) represents the aggregate interference from the active nodes on the horizontal road (resp. the vertical road), given by

$$I_{\text{X}} = \sum_{\mathbf{x}(d)\in\Phi_{\text{X}}\setminus\{\mathbf{u}\}} P_{\text{ul}} S_{\mathbf{x}} l(\|\mathbf{x}(d)\|), \quad \text{X} \in \{\text{H}, \text{V}\}, \quad (19)$$

where $S_{\mathbf{x}}$ denotes the fading for the transmitter at location $\mathbf{x}(d)$. For notational convenience we write $\mathbf{x}$ instead of $\mathbf{x}(d)$. To evaluate (18), we take the expectation with respect to both the useful signal power and the interference power, where the latter implicitly includes averaging over the spatial distributions of the interfering vehicles as well as the fading on the corresponding links. We write the success probability as

$$\mathbb{P}_{\text{ul}}^{\text{succ}}(d) = \mathbb{E}_{I_{\text{H}}, I_{\text{V}}}\left[\Pr\left\{S > \frac{\beta}{P_{\text{ul}} l(d)}(I_{\text{H}} + I_{\text{V}} + \sigma^2)\right\}\right]. \quad (20)$$

Using the complementary CDF of S, we find that

$$\mathbb{P}_{\text{ul}}^{\text{succ}}(d) = \exp\left(\frac{-\beta\sigma^2}{P_{\text{ul}} l(d)}\right) \times \mathbb{E}_{I_{\text{H}}, I_{\text{V}}}\left[\exp\left(\frac{-\beta}{P_{\text{ul}} l(d)} I_{\text{H}}\right) \exp\left(\frac{-\beta}{P_{\text{ul}} l(d)} I_{\text{V}}\right)\right]. \quad (21)$$

Due to the independence of the PPPs on the horizontal and vertical road, the success probability can be expressed as

$$\mathbb{P}_{\text{ul}}^{\text{succ}}(d) = \exp\left(\frac{-\beta\sigma^2}{P_{\text{ul}} l(d)}\right) \mathcal{L}_{I_{\text{H}}}\left(\frac{\beta}{P_{\text{ul}} l(d)}\right) \mathcal{L}_{I_{\text{V}}}\left(\frac{\beta}{P_{\text{ul}} l(d)}\right) \quad (22)$$

$$= \exp\left(\frac{-\beta N}{P_{\text{ul}} l(d)}\right) \left[\mathcal{L}_{I_{\text{H}}}\left(\frac{\beta}{P_{\text{ul}} l(d)}\right)\right]^2. \quad (23)$$

where $\mathcal{L}(\cdot)$ stands for the Laplace transform and the last transition is due to the symmetry of the interference around the center of the intersection. Considering a one-dimensional PPP, the Laplace transform originating from the horizontal road is given by

$$\mathcal{L}_{I_{\text{H}}}(\zeta) = \mathbb{E}[\exp(-\zeta I_{\text{H}})] \quad (24)$$

$$= \mathbb{E}\left[\prod_{\mathbf{x}\in\Phi_{\text{X}}\setminus\{\mathbf{u}\}} \exp(-\zeta P_{\text{ul}} S_{\mathbf{x}} (A\|\mathbf{x}\|)^{-\alpha})\right] \quad (25)$$

$$\stackrel{(a)}{=} \mathbb{E}_{\Phi_{\text{H}}}\left[\prod_{\mathbf{x}\in\Phi_{\text{X}}\setminus\{\mathbf{u}\}} \mathbb{E}_{S_{\mathbf{x}}}\{\exp(-\zeta P_{\text{ul}} S_{\mathbf{x}} (A\|\mathbf{x}\|)^{-\alpha})\}\right] \quad (26)$$

$$= \mathbb{E}_{\Phi_{\text{H}}}\left[\prod_{\mathbf{x}\in\Phi_{\text{X}}\setminus\{\mathbf{u}\}} \frac{1}{1 + \zeta P_{\text{ul}} (A\|\mathbf{x}\|)^{-\alpha}}\right] \quad (27)$$

$$\stackrel{(b)}{=} \exp\left(\int_{-\infty}^{+\infty} \frac{\Lambda(|x|)}{1 + (A|x|)^{\alpha}/\zeta P_{\text{ul}}} \mathrm{d}x\right) \quad (28)$$

$$\stackrel{(c)}{=} \exp\left(-\lambda/N_{\text{ul}} (\zeta P_{\text{ul}})^{\frac{1}{\alpha}} \frac{2}{A} \int_{\tilde{R}_{\text{a}}}^{+\infty} \frac{1}{1 + u^{\alpha}} \mathrm{d}u\right) \quad (29)$$

$$= \exp\Bigg(-2p\lambda/N_{\text{ul}} (\zeta P_{\text{ul}})^{\frac{1}{\alpha}} \tilde{R}_{\text{a}}^{1-\alpha} \times \frac{{}_2F_1\left(1, \frac{\alpha-1}{\alpha}, 2 - \frac{1}{\alpha}, -\tilde{R}_{\text{a}}^{-\alpha}\right)}{A(1-\alpha)}\Bigg), \quad (30)$$

where (a) holds due to the independence of the fading parameters, (b) uses the expression of the probability generating functional for a PPP [15, Definition A.5], (c) involves a change of variable $A|x|/(\zeta P_{\text{ul}})^{1/\alpha} \to u$, ${}_2F_1(\cdot)$ is the Gaussian hypergeometric function, and $\tilde{R}_{\text{a}} = R_{\text{a}}/(\beta^{1/\alpha} d)$. Note that even though the ego vehicle $\mathbf{u}$ belongs to Φ_{H} or Φ_{V} the results still hold due to Slivnyak's Theorem [15, Theorem A.5]. Specializing to the case $\alpha = 2$ and substituting $\zeta = \beta(Ad)^{\alpha}/P_{\text{ul}}$, (30) further simplifies to

$$\mathcal{L}_{I_{\text{H}}}\left(\frac{\beta(Ad)^2}{P_{\text{ul}}}\right) = \exp\left(-2d\frac{\lambda}{N_{\text{ul}}}\sqrt{\beta}\operatorname{arccot}\left(\frac{R_{\text{a}}}{\sqrt{\beta}d}\right)\right). \quad (31)$$

Substitution of (31) into (23) yields the desired result.

Appendix B
Proof of Proposition 2

The probability that two successive transmissions fail can be expressed as

$$\mathbb{P}_{\text{ul}}^{\text{fail}}(d, d - vT) = \mathbb{E}_{I_{\text{H}}, I_{\text{V}}}\left[\Pr\left\{S < \frac{\beta}{P_{\text{ul}} l(d)}(I_{\text{H}} + I_{\text{V}} + \sigma^2)\right\} \times \Pr\left\{\tilde{S} < \frac{\beta}{P_{\text{ul}} \tilde{l}(d)}(\tilde{I}_{\text{H}} + \tilde{I}_{\text{V}} + \sigma^2)\right\}\right] \quad (32)$$

where $\tilde{l}(d) = l(d - vT)$ and the notation $\tilde{\cdot}$ refers to random variables when the transmitter is at distance $d - vT$ from the intersection, and where we have used the independence of the small-scale fading at the successive time slots

$$\mathbb{P}_{\text{ul}}^{\text{fail}}(d, d - vT) = \mathbb{E}_{I_{\text{H}}, I_{\text{V}}}\left[(1 - A_1)(1 - A_2)\right] \quad (33)$$

where

$$A_1 = \exp\left(-\frac{\beta\sigma^2}{P_{\text{ul}} l(d)}\right) \exp\left(-\frac{\beta I_{\text{H}}}{P_{\text{ul}} l(d)}\right) \exp\left(-\frac{\beta I_{\text{V}}}{P_{\text{ul}} l(d)}\right) \quad (34)$$

and

$$A_2=\exp\left(-\frac{\beta\sigma^2}{P_{\mathrm{ul}}\tilde{l}(d)}\right)\exp\left(-\frac{\beta\tilde{I}_{\mathrm{H}}}{P_{\mathrm{ul}}\tilde{l}(d)}\right)\exp\left(-\frac{\beta\tilde{I}_{\mathrm{V}}}{P_{\mathrm{ul}}\tilde{l}(d)}\right). \quad (35)$$

Expansion of (33) yields

$$\begin{aligned}
&\mathbb{P}_{\mathrm{ul}}^{\mathrm{fail}}(d, d-vT)\\
&= 1-\mathbb{E}_{I_{\mathrm{H}},I_{\mathrm{V}}}[A_1]-\mathbb{E}_{I_{\mathrm{H}},I_{\mathrm{V}}}[A_2]+\mathbb{E}_{I_{\mathrm{H}},I_{\mathrm{V}}}[A_1A_2] \quad (36)\\
&= 1-\mathbb{P}_{\mathrm{ul}}^{\mathrm{succ}}(d)-\mathbb{P}_{\mathrm{ul}}^{\mathrm{succ}}(d-vT)+\mathbb{E}_{I_{\mathrm{H}},I_{\mathrm{V}}}[A_1A_2]. \quad (37)
\end{aligned}$$

This last term can be written as

$$\begin{aligned}
\mathbb{E}_{I_{\mathrm{H}},I_{\mathrm{V}}}[A_1A_2] &= \exp\left(-\frac{\beta\sigma^2}{P_{\mathrm{ul}}}\left(\frac{1}{l(d)}+\frac{1}{\tilde{l}(d)}\right)\right) \quad (38)\\
&\times\mathbb{E}_{I_{\mathrm{H}}}\left[\exp\left(-\frac{2\beta}{P_{\mathrm{ul}}}\left(\frac{I_{\mathrm{H}}}{l(d)}+\frac{\tilde{I}_{\mathrm{H}}}{\tilde{l}(d)}\right)\right)\right]
\end{aligned}$$

where we have made use of the identical distribution of the interference on both roads. After substitution of (19), the last factor in (38) can now be expressed as

$$\begin{aligned}
&\mathbb{E}_{I_{\mathrm{H}}}\left[\exp\left(-\frac{2\beta}{P_{\mathrm{ul}}}\left(\frac{I_{\mathrm{H}}}{l(d)}+\frac{\tilde{I}_{\mathrm{H}}}{\tilde{l}(d)}\right)\right)\right] \quad (39)\\
&= \mathbb{E}_{I_{\mathrm{H}}}\Bigg[\prod_{\mathbf{x}\in\Phi_{\mathrm{X}}\setminus\{\mathbf{u}\}}\exp\left(\frac{-2\beta S_{\mathbf{x}}(A\,|x|)^{-\alpha}}{l(d)}\mathbb{I}\{|x|>R_{\mathrm{a}}\}\right)\\
&\times\exp\left(\frac{-2\beta\tilde{S}_{\mathbf{x}}(A(|x|-vT))^{-\alpha}}{\tilde{l}(d)}\mathbb{I}\{|x|>R_{\mathrm{a}}-vT\}\right)\Bigg],
\end{aligned}$$

in which x is the first component of $\mathbf{x}$ and $\mathbb{I}\{\cdot\}$ is the indicator function. Taking the expectation over the fast fading, which is independent from vehicle to vehicle and from time slot to time slot, we find that

$$\begin{aligned}
&\mathbb{E}_{I_{\mathrm{H}}}\left[\exp\left(-\frac{2\beta}{P_{\mathrm{ul}}}\left(\frac{I_{\mathrm{H}}}{l(d)}+\frac{\tilde{I}_{\mathrm{H}}}{\tilde{l}(d)}\right)\right)\right] \quad (40)\\
&= \mathbb{E}_{\Phi_{\mathrm{X}}}\Bigg[\prod_{\mathbf{x}\in\Phi_{\mathrm{X}}\setminus\{\mathbf{u}\}}\left[\frac{1}{1+2\beta(A\,|x|)^{-\alpha}/l(d)}\right]_{R_{\mathrm{a}}}\\
&\times\left[\frac{1}{1+2\beta(A(|x|-vT))^{-\alpha}/\tilde{l}(d)}\right]_{R_{\mathrm{a}}-vT}\Bigg],
\end{aligned}$$

in which we introduced the the notation

$$[f(x)]_a=\begin{cases} f(x) & |x|\geq a\\ 1 & \text{else.}\end{cases} \quad (41)$$

Further introducing

$$\begin{aligned}
g(x) &= \left[\frac{1}{1+2\beta(A\,|x|)^{-\alpha}/l(d)}\right]_{R_{\mathrm{a}}} \quad (42)\\
&\times\left[\frac{1}{1+2\beta(A(|x|-vT))^{-\alpha}/\tilde{l}(d)}\right]_{R_{\mathrm{a}}-vT}
\end{aligned}$$

and by using the probability generating functional for a PPP, in a similar manner as in (28), we find that

$$\begin{aligned}
&\mathbb{E}_{I_{\mathrm{H}}}\left[\exp\left(-\frac{2\beta}{P_{\mathrm{ul}}}\left(\frac{I_{\mathrm{H}}}{l(d)}+\frac{\tilde{I}_{\mathrm{H}}}{\tilde{l}(d)}\right)\right)\right] \quad (43)\\
&= \exp\left(-2\frac{\lambda}{N_{\mathrm{ul}}}\int_0^{+\infty}(1-g(x))\,\mathrm{d}x\right),
\end{aligned}$$

which can be evaluated numerically. Substitution of (43) into (38) yields the desired result.

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